The Karlsruhe Physics Course

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Received 22 July 1999

Abstract. The Karlsruhe Physics Course is an attempt to modernize the physics syllabus by eliminating obsolete concepts, restructuring the contents and extensively applying a new model, the substance model. The course has been used, tested and improved for several years, and we believe that the time has come to make it known to a wider public. We introduce the structure which underlies the course and discuss some consequences for the teaching of various subfields of physics.

1. Introduction

The amount of physics knowledge is increasing steadily, whereas the time available to teach the subject remains essentially unchanged. This fact obviously requires a continuous effort of adjustment and reprocessing of our knowledge. In this paper we report on one such effort: a physics course which has been developed over the past 20 years at the Didactics Department of the University of Karlsruhe. Although the most elaborate version of the course is for the lower secondary school (Herrmann 1994, 1997, 1998a), the project was not just to write a new schoolbook. The objective was to develop a new way of teaching physics, independent of the target group of learners. The course is now known as the Karlsruhe Physics Course.

Although the course contains innovations in many details, it can be said that its peculiarities are essentially based on three ideas:

1. The historical development of physics has followed an intricate path. When teaching, we have in the past imposed this path on our students, although there are shorter and easier ways of achieving the same goals (Herrmann and Job 1994–1998, 1996). We have tried to eliminate such historical burdens from the physics syllabus.

2. We have chosen a unified approach to science teaching, based upon a certain class of quantities that play a fundamental role in classical and modern physics: the extensive quantities energy, momentum, angular momentum, electric charge, amount of substance and entropy. When emphasizing extensive quantities, the subdivision of physics into sub-branches is nothing more than a classification of natural processes according to the extensive quantity playing the dominant role in each case. Knowledge of a single branch of physics already provides an analogy for the means by which processes are described in other branches of science, including chemistry and biology (Herrmann 1998b).

3. We make extensive use of a model which, in the traditional curriculum, plays only a minor role, namely the substance model.

In the past few years this course had considerable impact, and other workers have continued and extended this line of work. An example is a university text book about thermodynamics by Fuchs (1996).
In no way do we want to claim that our proposal is a kind of definite solution. On the contrary, apart from presenting our own work, we would like to stimulate a debate about this approach and to encourage other efforts to drastically reduce the size of the physics syllabus.

In section 2 we shall comment about the manner in which our project was carried out, and about the origin of the ideas underlying the course. In section 3, the structure of the course is presented. In section 4 the substance model is introduced and section 5 contains some of the specific consequences for various sub-fields as treated in the course.

2. General remarks

2.1. The project

All the parts of the course have been developed in essentially the same way. The first step was always to work on a restructuring of the subject, independently of the level of student at whom it might finally be aimed. Next, a version of a concrete course was developed for physics students at the University of Karlsruhe (Herrmann 1997). We never began with an elementary version: otherwise, one could not be sure that the result had sufficiently strong foundations to support the more advanced versions.

Only after having completed the university course were more elementary versions developed. The greatest amount of time was invested in the development of a course for the lower secondary school. The first applications and tests of this high-school version have been developed by ourselves, i.e. myself and other members of our University Working Group. For that purpose we were permanently teaching at a German High School. After several test runs at this school and after a complete written version had been developed, about 20 other professional school teachers took over the test work. Feedback was given to us in order to improve the new editions of the course. Up to the time of writing an estimated 8000 pupils have completed the programme. The tests were strictly supervised by the school authority. During three years of intense observation no major difficulties arose, and the school authorities gave their approval for the general use of the concept in the High Schools of the Land (Federal State) of Baden-Württemberg. They also changed the teaching programme in such a way that the Karlsruhe Physics Course can be used as an alternative to the traditional schoolbooks†.

2.2. The origins of the course

The Karlsruhe Physics Course contains numerous differences compared with traditional physics courses. However, there is no innovation which does not have its roots in some previous research, sometimes in the ideas of renowned scientists. We shall cite such sources in the following sections where appropriate. There are a few sources, however, which we would like to emphasize, and we shall mention these in advance.

1. We owe our knowledge about the analogy, which is so important for the conciseness of the course, to Falk’s axiomatic treatment of thermodynamics (Falk 1968, Falk and Ruppel 1976, Falk 1990, Falk and Herrmann 1977–1982). Falk’s work, on the other hand, can be seen to have its roots in the thermodynamics of Gibbs (1961). Essentially, the same analogy is also applied in many typical text books about the thermodynamics of irreversible processes: the emphasis on currents and forces, where ‘force’ is taken to mean a gradient of the energy-conjugated intensive variable (see, for instance, Callen 1985). The same analogy is also used by other authors as a basis of what is often called system physics (Burkhardt 1987) or system dynamics (Fuchs 1996, Maurer 1990, 1998).

2. The study of mechanics, which is based on the interpretation of forces as momentum currents, has a tradition that is relatively young compared with the respectable age of classical mechanics. The proposal to interpret a force as a momentum flow was first made by Max Planck in 1908 (Planck 1908). This date is not accidental. With the publication of the theory of relativity three years earlier, it became clear that energy and momentum should be understood as basic quantities, and not as being derived from supposedly more fundamental quantities like mass, velocity, and force. However, the considerable age and dignity of classical mechanics has prevented this up-to-date and simpler interpretation of forces from gaining admission to elementary text books. As far as we know, the first efforts in this respect are due to Di Sessa (1980) and to ourselves (Herrmann 1979).

In advanced texts, in particular in texts about hydrodynamics, this interpretation has been customary since its first publication by Planck: see, for instance, Pauli (1963) or Landau and Lifshitz (1959).

3. We owe the idea that the common-language concept of heat perfectly matches the properties of the physical quantity entropy to the work of Job (1972). Job himself, when he published his *Neudarstellung*, was still unaware that the same idea had been presented with great clarity much earlier in a paper by Callendar (1911), which had apparently since been forgotten.

4. Our treatment of the chemical potential as a universal driving force for all those processes where the quantity ‘amount of substance’ is created, destroyed, or transported is also due to Job (1978, 1981a, b).

2.3. System dynamics modelling tools

At the same time as we were developing our physics with substance-like quantities, a new type of computer software was developed and improved, which fits perfectly into this kind of physics, this sometimes being referred to as system dynamics modelling tools. Examples of such programs are *Stella* (High Performance Systems, Inc., Hanover, NH, USA), *Powersim* (Powersim AS, Isdalsto, Norway), and *Dynano* (Pugh-Roberts Associates, Cambridge, MA, USA).

Modelling software is now considered by many teachers and school authorities as an important learning tool, and may soon find a place in many syllabuses.

3. The structure of the course

3.1. Substance-like quantities

In the Karlsruhe Physics Course, a certain class of physical quantities plays the role of basic concepts: the extensive or, as we like to call them, substance-like quantities (Falk 1968, 1977). Among the substance-like quantities are mass, energy, electric charge, amount of substance, momentum, angular momentum, and entropy. Each extensive quantity \( X \) obeys a continuity
equation, which, in its integral form, reads
\[
\frac{dX}{dt} = I_X + \Sigma_X. \tag{1}
\]
The validity of such an equation allows us to interpret \(X\) as a measure of the amount of a substance or fluid, \(I_X\) as a current intensity of \(X\) (Herrmann 1986), and \(\Sigma_X\) as the production rate of \(X\). By ‘interpret’ we mean that we are applying a model when dealing with these quantities, the ‘substance model’.

According to this model, the change of the value of \(X\) has two causes. On the one hand a production or destruction of \(X\) within the region of space considered, and on the other a flow through its boundary surface. Thus, equation (1) establishes a balance of the quantity \(X\). Equation (1) can be transformed into a local form:
\[
\frac{\partial \rho_X}{\partial t} = \text{div} j_X + \sigma_X. \tag{2}
\]
Here, \(\rho_X\) is the density of \(X\), \(j_X\) the current density and \(\sigma_X\) the production density.

For some substance-like quantities, the term \(\Sigma_X\), or \(\sigma_X\), is always equal to zero. These quantities can change their value within the region only by a flow through the boundary surface. They are called ‘conserved quantities’. Energy and electric charge are examples of conserved quantities. Accordingly, for electric charge equation (1) reads
\[
\frac{dQ}{dt} = I. \tag{3}
\]
Here, \(I\) is the strength of the electric current. For the energy we obtain
\[
\frac{dE}{dt} = P \tag{4}
\]
where \(P\) is the strength of the energy current or ‘power’.

Other substance-like quantities, such as entropy and amount of substance, can change their value by production and/or destruction. Thus, a substance-like quantity is not necessarily a conserved quantity.

A substance-like quantity must not be scalar. Momentum and angular momentum are examples for vectorial substance-like quantities. If the coordinate system is kept fixed, one is allowed to imagine a vectorial substance-like quantity as three scalar substance-like quantities, where for each of the three components a balance equation of the form (1) holds.

### 3.2. The analogy

When using the extensive quantities as a basis for structuring the course, one can take advantage of a far-reaching analogy between the various parts of physics, and also physical chemistry if the quantity ‘amount of substance’ is added to the list of substance-like quantities.

According to table 1 the extensive quantities electric charge \(Q\), momentum \(p\), entropy \(S\) and amount of substance \(n\) correspond to each other. The same holds true for the conjugate

<table>
<thead>
<tr>
<th>Extensive quantity</th>
<th>Conjugate intensive quantity</th>
<th>Current</th>
<th>Subfield of science</th>
<th>Energy flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric charge (Q)</td>
<td>Electric potential (\phi)</td>
<td>Electric current (I)</td>
<td>Electricity</td>
<td>(P = UI)</td>
</tr>
<tr>
<td>Momentum (p)</td>
<td>Velocity (v)</td>
<td>Force (F)</td>
<td>Mechanics</td>
<td>(P = v \cdot F)</td>
</tr>
<tr>
<td>Entropy (S)</td>
<td>Absolute temperature (T)</td>
<td>Entropy current (I_S)</td>
<td>Thermodynamics</td>
<td>(P = TI_S)</td>
</tr>
<tr>
<td>Amount of substance (n)</td>
<td>Chemical potential (\mu)</td>
<td>Substance current (I_n)</td>
<td>Chemistry</td>
<td>(P = \mu I_n)</td>
</tr>
</tbody>
</table>
intensive quantities electric potential $\phi$, velocity $v$, absolute temperature $T$ and chemical potential $\mu$. For each of the extensive quantities a flow or current exists: the electric current $I$, the momentum current or force $F$ (Herrmann and Schmid 1984, 1985a, Herrmann et al. 1987), the entropy current $I_S$ and the substance current $I_n$.

Many of the relationships that exist between the quantities of one subfield of science (one row in the table) have a counterpart in another subfield. An example is shown in the last column of table 1. Each of the equations in this column represents a description of an energy transport. It is customary to say that energy is transmitted in one or the other ‘form’, according to which of the equations describes the transmission (Falk 1968). The first equation ($P = UI$) corresponds to the so-called electric energy. (The letter $U$ stands for an electric potential difference.) If the pertinent relation is that of the second row, then the energy exchange is called ‘work’. The equation $P = TIS$ (third row) describes transport in the form of heat and $P = \mu I_n$ (fourth row) corresponds to chemical energy.

There are quantities which have to be translated into themselves when applying the analogy of table 1. On the one hand the kinematic quantities time and position, and on the other the energy. Thus, of the substance-like quantities, the energy plays a prominent part. The energy is not characteristic of only one or some of the subfields of physics, rather it is equally important in all of them. It plays the role of a unifying concept.

4. The substance model

The particle model is probably the best-known and most successful model of classical physics. However, of the models currently in use, there is another, which also has a long history, but which never received the same kind of recognition as the particle model. It does not even have an established name. We propose to call it the ‘substance model’. As this name suggests, some physical object is imagined to be a fluid or substance or, in more colloquial terms, ‘stuff’. Here, the concept of substance is to be understood in its common language meaning. An old example for the application of this model is to picture the electric charge as a substance or fluid. Hence, the name electric ‘current’ for the quantity $I$.

We propose to make extensive use of this model, and not to limit it to those cases where it has traditionally been employed. We shall discuss three very distinct modes of application of the substance model. The first application is in comparing extensive physical quantities with substances. The second is that fields are considered as substances, and in our third application we interpret what is currently called ‘the probability density of finding a particle at a point’ as the ‘density of a substance’.

Note that when comparing a certain amount of an extensive quantity, a portion of the electromagnetic field or a certain amount of the square of the wavefunction with a substance, we do not claim that these entities are substances. All we do is to apply a model. A model, however, is never correct or false. A model can only be more or less appropriate.

4.1. Physical quantities in the substance model

When introducing a new physical quantity the student has to learn, among other things, the verbal ‘environment’ of the quantity: verbs, adjectives, adverbs and prepositions which go together with the quantity. When formulating sentences with the quantities ‘force’, ‘work’ or ‘electric potential difference’ there is not much freedom: a force ‘acts on a body’ or is ‘exerted by a body’, work is ‘done’ or ‘performed’ and an electric potential difference ‘exists’ or ‘is applied’.

In contrast, when dealing with ‘substance-like’ quantities it is correct to use all those figures of speech which are used when speaking about a substance. Thus, it is appropriate to say, that a body contains a certain amount of electric charge, but also that the charge sits on the body. Electric charge can flow from one place to another. It can be accumulated, concentrated, diluted, distributed, lost, collected and much more. The explanation of where
the great freedom in the use of language comes from is that we are applying the substance model. Every student is already familiar with this language before his or her first science lesson. Emphasizing the substance-like character of these quantities represents a substantial aid when teaching science.

In the traditional teaching of science one does not always take advantage of this possibility. Usually, the substance model is only applied when dealing with the quantities mass and electric charge. Energy, entropy and momentum, on the other hand, are derived from other quantities. By doing so, the insight that they are substance-like is obscured.

The substance model for extensive quantities can be extended in such a way that it also includes the corresponding intensive quantities. Let us explain this claim, again with the example of electricity. Suppose an electric current flows through what is called a resistor. The word resistor already belongs to the substance model. The word suggests that this device opposes the flow of electricity. In order to flow, a kind of driving force is needed, an electric potential difference or electric tension. Thus, the potential difference appears as the cause, and the electric current as the effect. Let us show the arbitrariness of this wording with the help of the special case of Ohm’s law:

\[ U = RI. \]  

(5)

The equation tells us that \(U\) and \(I\) are proportional to one another. It does not tell us anything about which quantity is the cause and which is the effect. We feel it more natural to call the potential difference the cause, but the reason is simply that in most cases on a power supply we adjust the voltage and not the current. Indeed, when running the power supply in current-stabilized mode, it is more natural to say that the current causes a potential drop: the current is the cause, the potential difference the effect.

In spite of its arbitrariness, the substance model has proved to be extraordinarily useful for the teaching of electricity. The learner can orient himself by means of those phenomena from which the model stems, namely currents of water or other fluids.

However, the real power of the model comes from the fact that it applies not only to electricity but also to other extensive quantities. Just as an electric potential difference can be interpreted as a driving force for a current of electric charge, a temperature difference can be considered as the cause of an entropy current, a chemical potential difference is necessary to drive a substance current and a velocity difference is responsible for a momentum current. (The momentum which a car loses due to friction increases with the speed of the car.) There is a great advantage in terms of teaching economy when using the picture not only for electricity but also for other dissipative transport phenomena.

Finally, we propose one further extension of the substance model. The equations in the last column of table 1 suggest a simple picture for the description of an energy transport. We call the substance-like quantity which is flowing simultaneously with the energy according to table 1, the energy carrier (Falk and Herrmann 1981, Falk et al 1983). Thus, the energy is carried by momentum, electric charge, entropy or amount of substance. In a device which in traditional terms is called an energy converter the energy simply changes its carrier. It enters with one carrier, is then ‘trans-shipped’ to a second carrier and leaves the device with this new carrier.

When teaching mechanics or thermodynamics traditionally, one encounters problems which are said to be related to false preconceptions or misconceptions of the students. Instead of changing the students’ ideas such that they conform to the way physics is presented, one could rather think about finding a presentation of physics that conforms better to the way people reason in everyday-life situations. The use of the substance model has turned out to be a step in this direction.

4.2. Fields in the substance model

Usually, fields are introduced as very particular creatures. It is said that a field is a region of space with certain properties, or that an electric field is the space around a charged body.
Actually, this way of speaking evokes the impression that the concept of a field is rather mysterious. It is suggested that a field is a portion of empty space with certain properties or, even worse, a kind of nothing with properties.

Actually, there is no reason for such mystification. A field is a physical system. As any other physical system, it can admit various states and in each state the physical standard variables have certain values (Falk 1968, p 54, Herrmann 1989). Just as any other physical object, an electromagnetic field has energy, momentum, entropy and pressure (or mechanical stress). In certain states it has a temperature and a chemical potential. When speaking about a field, there is no reason to apply different wording to when speaking about a material system, a gas for instance.

In order to explain to somebody what air is, it would be perfectly correct to say air is a portion of space with certain properties, but it would not be a very meaningful explanation. The same holds true for a field. Instead of introducing a field as a portion of space with certain properties, it is clearer to say a field is an object with these properties. Actually, when speaking in this way, we are employing the substance model.

In order to explain to somebody what air is, it would be perfectly correct to say air is a portion of space with certain properties, but it would not be a very meaningful explanation. The same holds true for a field. Instead of introducing a field as a portion of space with certain properties, it is clearer to say a field is an object with these properties. Actually, when speaking in this way, we are employing the substance model.

However, in order to fully profit from this model, we have to complement the language. We need one name for the object under consideration and another name for the material the object is made of. For the first concept a name already exists, namely field. For the second, in traditional physics there is no name. As a provisional name, in the Karlsruhe Physics Course we use the word ‘field stuff’ (*Feldstoff* in German).

4.3. Electrons in the substance model

Atoms are usually introduced as consisting of a nucleus and an electron shell. This introduction mostly begins with the creation of a cognitive conflict. It is said that an electron is a small object which moves around the nucleus, but which does not have a trajectory. To avoid this conflict we have chosen a model in which the electron does not move as long as its state is an eigenstate of the energy. According to this model, the nucleus is surrounded by a substance which we call electronium. The density of the electronium is what normally is called the electron density or the probability density of finding the electron at the point under consideration, i.e. for one-electron systems the square of the wavefunction. That density of the electronium is what is currently determined by means of x-ray diffraction techniques. According to this model, an electron is an elementary portion of the electronium: a portion with a charge of $-1.6 \times 10^{-19}$ C.

Thus, an electron, although having a definite charge and mass, does not possess a unique shape and extent. Its shape is the same as that of the square of the wavefunction and thus depends on the state of the electron.

5. The course

In this section, some of the consequences of the general ideas described so far for the concrete design of the course are outlined. However, it is not a summary of the contents of the course.

5.1. Mechanics

According to our basic structure, momentum is the central quantity of mechanics. It can be used to define what mechanics is: that part of physics which deals with momentum and its currents (and later with angular momentum and its currents). Therefore, it is natural to introduce momentum at the very beginning of the mechanics part. It is introduced operationally by a measuring method (Herrmann and Schubart 1989). For an intuitive understanding, momentum is introduced as a measure of the content of movement of a body, what in colloquial terms would be called the ‘force’, the ‘power’ or the ‘impetus’ of a body†. Since it is a fundamental

† Actually, all of our teaching experience was done with German pupils. The common language words which they proposed were typically: ‘Schwung’, ‘Kraft’, ‘Wucht’ and the English word ‘Power’ – a word which is well known
quantity we give a proper name to its SI unit: 1 N s is called 1 Huygens (abbreviated Hy).

Momentum can go or flow from one body to another. Spontaneously, i.e. in a frictional or dissipative process, it flows from the body with a higher velocity to the one with the lower velocity. In order to get it in the direction opposite to this natural tendency one needs what we like to call a momentum pump. Often, a motor is used as a momentum pump.

Newton’s laws, when formulated with momentum currents, reveal themselves to be no more than the expression of momentum conservation. Since the conservation of momentum is presupposed to be valid from the beginning (just as in electricity the conservation of the electric charge is currently taken for granted), a formulation of Newton’s laws is no longer necessary (Herrmann and Schmid 1985b).

In our approach friction is not an unwanted phenomenon, while it is viewed only as a nuisance in the realm of pure Newtonian mechanics. It is a most natural process, in which momentum passes dissipatively from one body to another. Thus, the rectilinear movement of a car on a road appears as a steady state or flow equilibrium: the momentum which its motor is steadily pumping into the car is equal to the momentum flowing out into the air and into the ground due to friction processes.

5.2. Thermodynamics

Just as mechanics begins with momentum, thermodynamics begins with the introduction of entropy. The intuitive understanding which we are stimulating is that entropy is what in colloquial terms would be called heat or amount of heat. (Callendar 1911, Job 1972, Falk 1985, Fuchs 1986, 1987, 1996). Just as with momentum, entropy also gets its own SI unit: 1 J K$^{-1}$ is called 1 Carnot (abbreviated to Ct). This is an old proposal by Callendar (1911). It is stated that a body at zero Kelvin has an entropy content of 0 Ct, or in more colloquial terms: an absolutely cold body does not contain heat. Entropy can be produced. It is produced in processes which represent a kind of friction: mechanical friction, electric ‘friction’ in an electric resistor, or chemical ‘friction’ in a free-running chemical reaction. The second principle is formulated in the following way: entropy can be produced but not destroyed.

Entropy flows spontaneously from a body of higher to a body of lower temperature. Since this is also a kind of frictional process, new entropy is produced in this process.

In order to get an entropy flow in the direction opposite to its natural tendency one needs an entropy pump, technically called a heat pump.

5.3. Electricity

The structure of electricity did not experience major changes in our course since in the traditional courses the substance model is already used. Electric charge flows spontaneously from places of higher to places of lower electric potential, and in order to get it flowing against this natural tendency an ‘electricity pump’ is necessary: a battery or a generator or a solar cell.

The magnetic field is then introduced as a very concrete thing which sticks to magnetic poles and to current-carrying conductors.

5.4. Reactions

The course also contains a chapter about physical chemistry and this chapter has also the same structure as mechanics, heat and electricity (Job 1978, 1981a, b). Just as an electric potential difference acts as a driving force of an electric current, a chemical potential difference causes a chemical reaction to run, to make a substance change its phase or to drive a diffusion current.

to German youngsters. Notice that the word ‘Kraft’ (in English ‘force’) has, in common language, a much wider meaning than in physics. Moreover, the correspondence with the physical concept of momentum seems to be better than that with the physicist’s force.
In analogy with electrical resistance, we introduce a chemical or reaction resistance. A catalyst then appears as a switch which allows one to turn a chemical reaction on and off.

The fuel cell and the electrolytic cell are introduced as devices in which energy is transferred from the energy carrier ‘amount of substance’ to the energy carrier ‘electricity’ and vice versa.

5.5. Data

We have shown that in our course the energy plays the role of a structural concept. There is yet another quantity with this ability to provide structure, namely Shannon’s ‘amount of data’. It is the unifying quantity of all those parts of science and technology which have to do with the transmission, processing and storage of data: optics, acoustics, electronics, computer science. Just as with the analogy discussed previously, the analogy between energy and amount of data has a sound physical basis (Herrmann and Schmid 1986).

It is important that the rapidly developing domain of data transport, processing and storage does not appear as an appendix to electronics or as a sub-branch of mathematics, as is the case in various other teaching programmes.

In our course, energy transmissions are classified according to the respective energy carrier. In the same manner we classify data transmissions according to the pertinent data carrier (Herrmann et al 1985, Herrmann and Schmälzle 1987).

6. Conclusions

The physics syllabus needs a continuous effort of modernization. In our opinion, physicists have in the past neglected this task to some extent. That is why the need for major changes has accumulated. With the Karlsruhe Physics Course we propose an example of such a change.

Although many students have gone through the course by now, we are still not able to judge the results. An evaluation of the course for the lower secondary school based on a sample of about 600 pupils is now under way (Starauschek 1998). However, much more has to be done in order to know the consequences.

With the present paper we would like to motivate a wider public to participate in the discussion of this proposal, as well as encourage work on other alternatives.

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